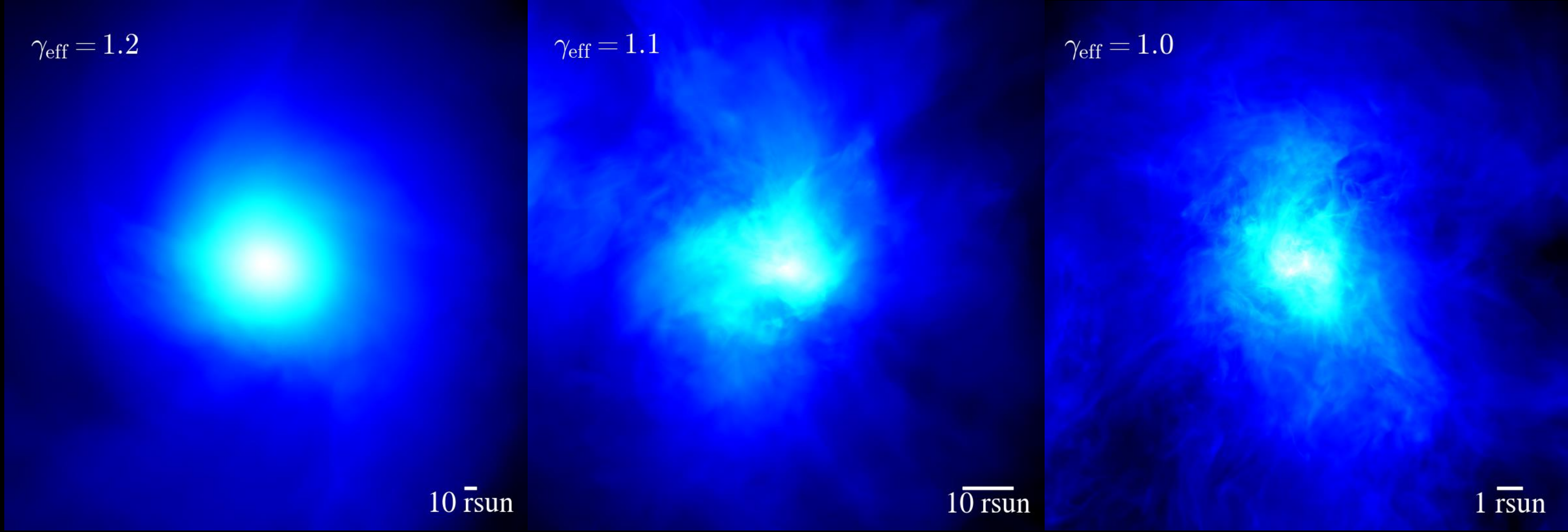

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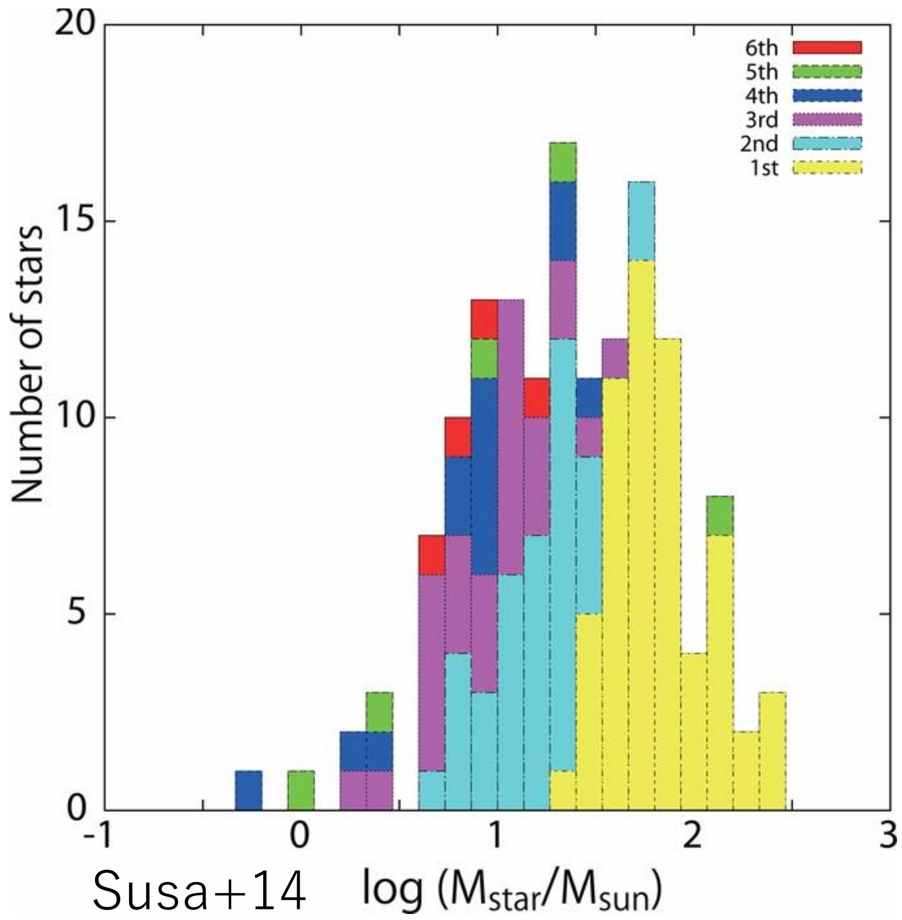
Amplification and saturation of turbulent magnetic field in collapsing primordial clouds

2023.11.20

Importance of IMF of the first stars



Susa+14

Theoretical predictions

If Pop III mass M is

$\leq 0.8\ M_{\odot}$
Can survive to present

$8 \leq M \leq 40\ M_{\odot}$
Core-collapse supernovae

$40 < M < 140\ M_{\odot}$
Direct collapse black hole

$140 M_{\odot} \leq M \leq 260\ M_{\odot}$
Pair-instability supernovae

Need to match

◆Pop III survivors in Milky way

◆EMP stars$(10^{-5} - 10^{-3} Z_{\odot})$

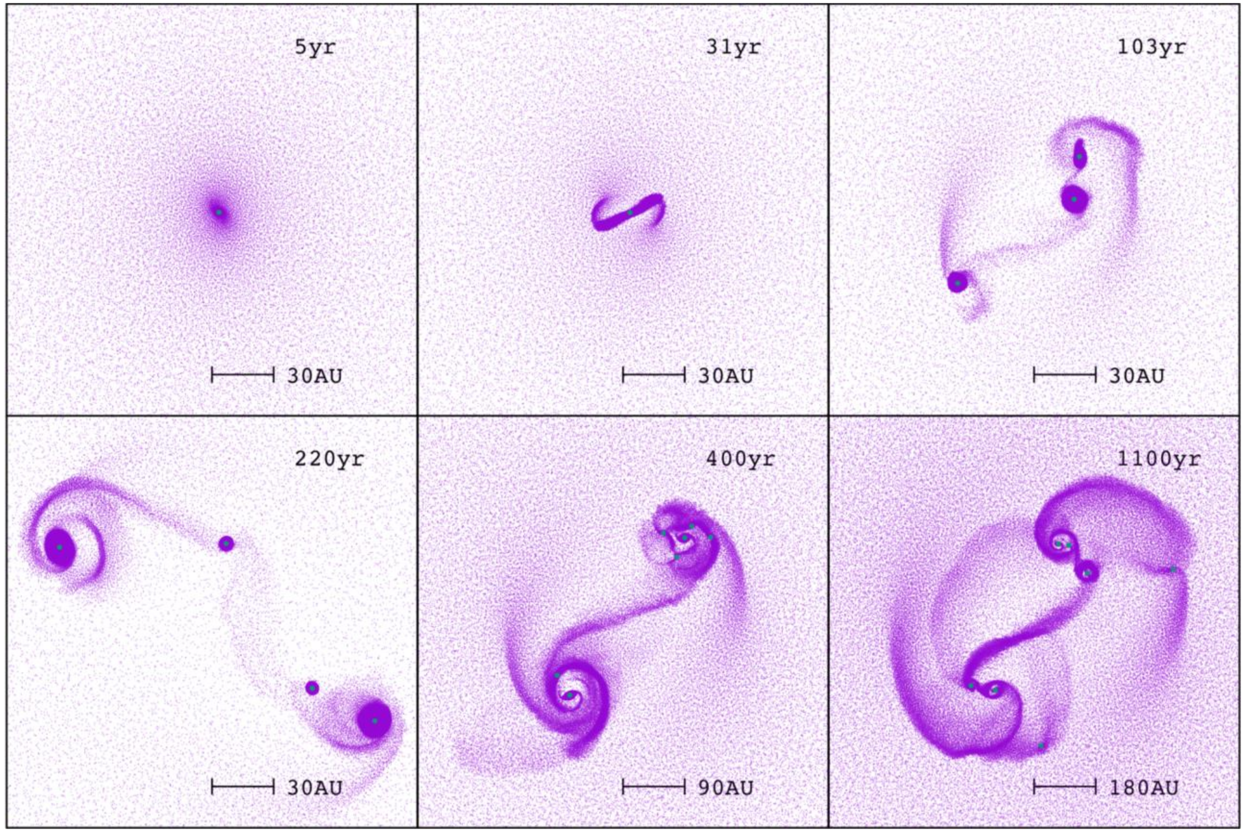
◆GW source

◆Special abundance of EMP stars$(< 10^{5} Z_{\odot})$

(Metallicity range have been shown by Chiaki+18)

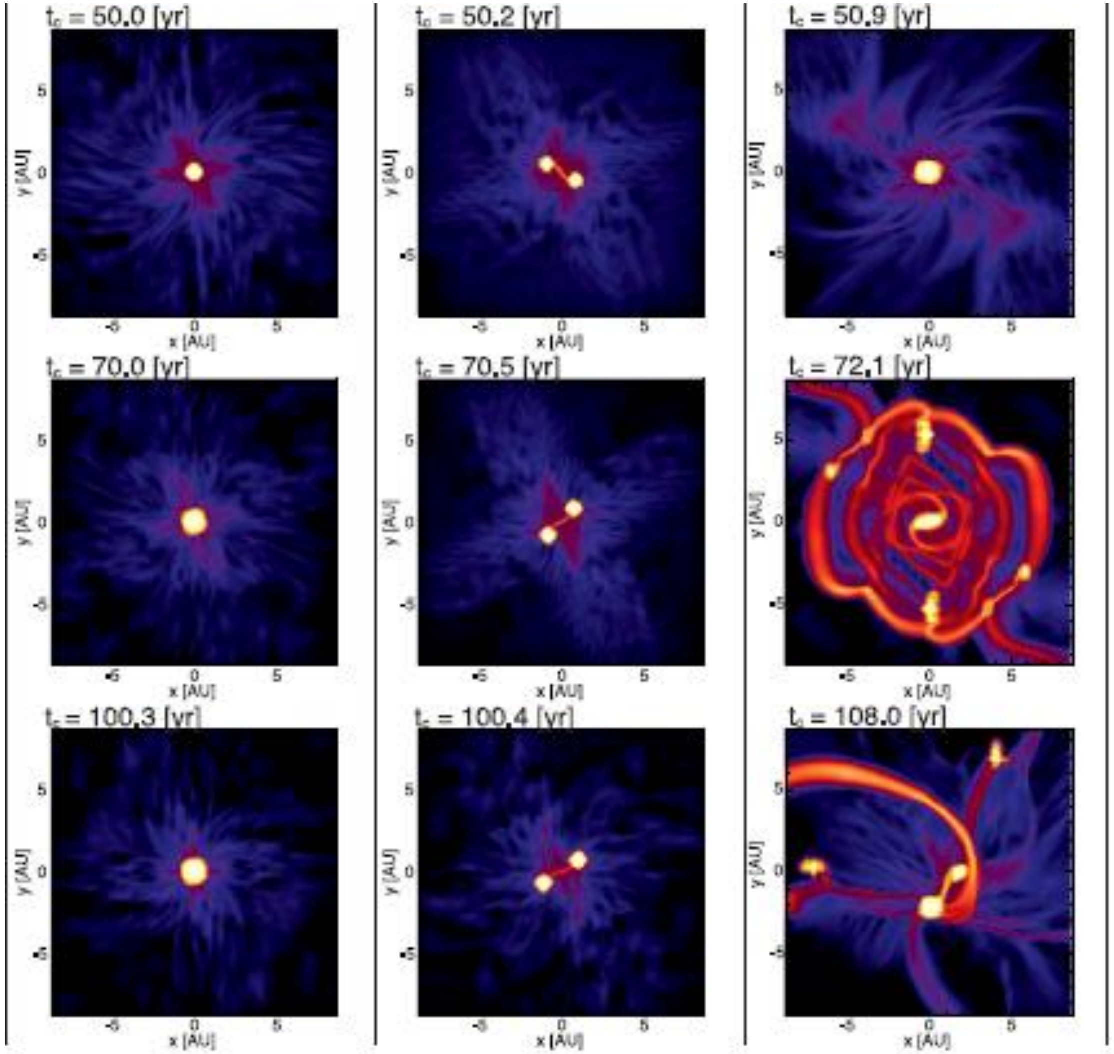
Observations

Disk fragmentation after protostar formation



Susa 2019 (hydro simulation)

←(strong) B-field strength (weak)→

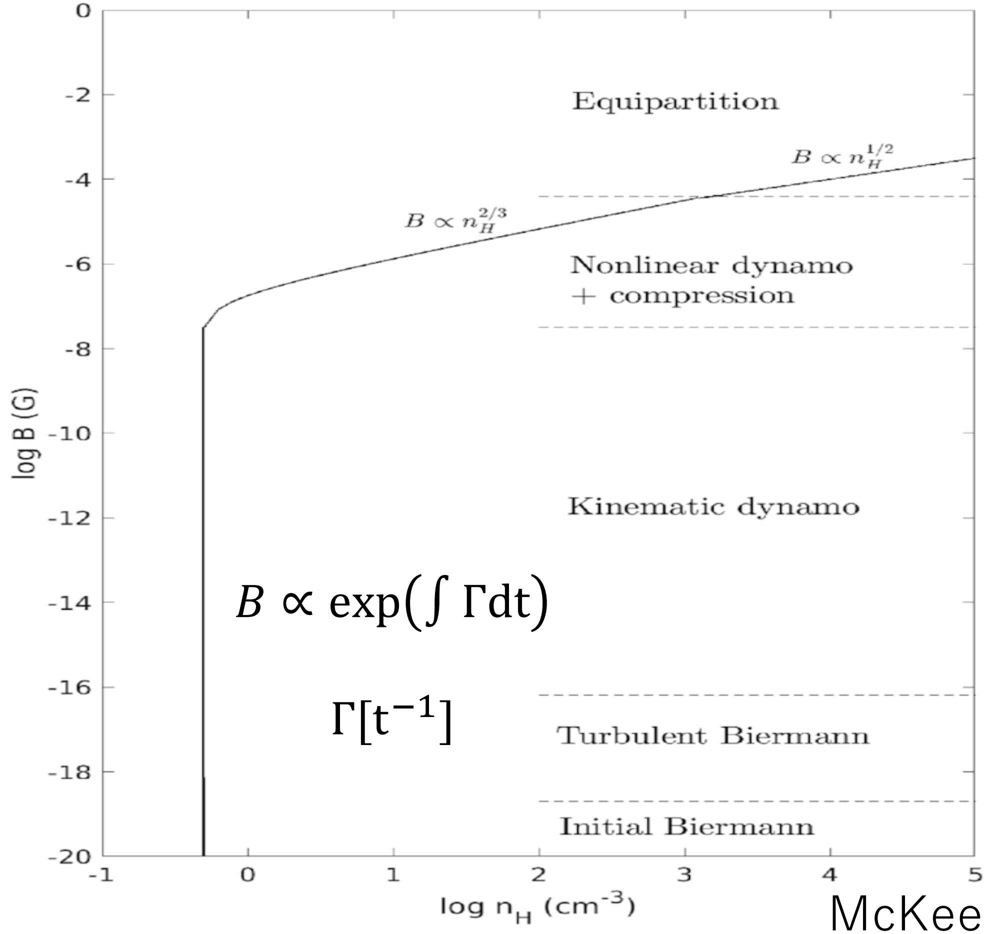


Disk fragmentation influences diversity in the mass spectrum.

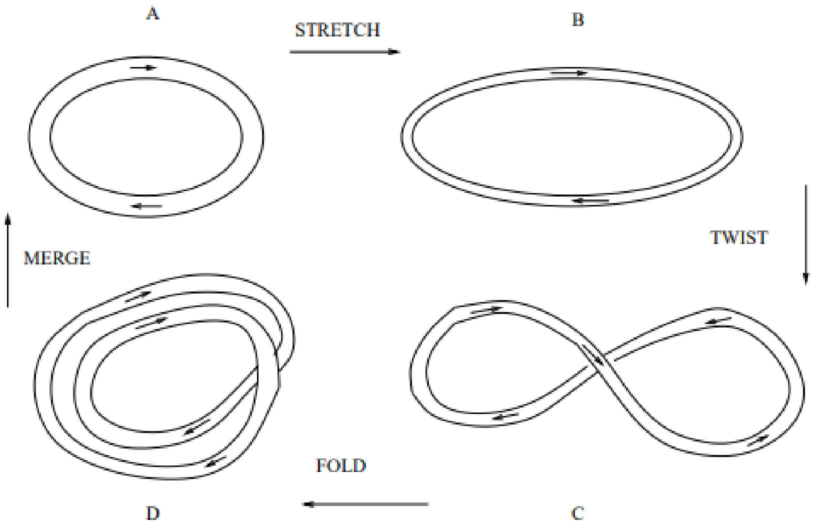
Strong B-field generally suppresses the fragmentation.

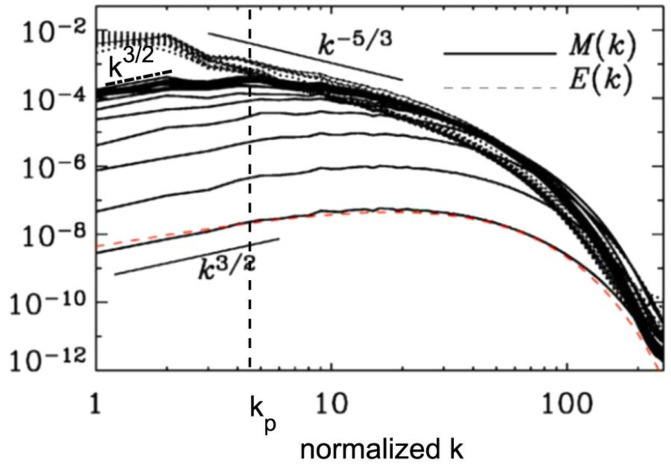
Machida&Doi 13

Magnetic field in the early universe



McKee & Stacy 20





Brandenburg & Subramanian 05

- ✓ **Driven on a small scale and diffused to a large scale**
- ✓ **The initially weak field is amplified until the equipartition level of the turbulence.**

Earlier analytic/theoretical studies for the magnetic field growth

Kinematic stage
($\varepsilon_B \ll \varepsilon_{\rm turb}$ on viscous scale)

$$B_{\rm kin} = B_0 \xi^{2/3} \exp\left(\frac{3}{8}\int \Gamma_\nu \, dt\right),$$

Flux-freezing
(flux conservation)

$\xi \equiv \rho/\rho_0$

Exponential growth due to rapid turnover of the eddy on ν scale

where $\int \Gamma_\nu dt = (3/32)^{1/2} \mathrm{Re}^{1/2} \langle \mathcal{M}_{\rm turb} \rangle \int (1/t_{\rm ff}) dt$

Non-linear stage
($\varepsilon_B \sim \varepsilon_{\rm turb}$ on viscous scale)

$$B_{\rm nl} = B_1 \mathcal{A}_1 \left(\frac{\xi}{\xi_1}\right)^{(1+a)/2}$$

Flux-freezing is violated by reconnection diffusion

where $\mathcal{A}_1^2 = 1 + 0.013 \frac{\phi_{\rm ff}\mathcal{M}_{\rm turb}}{a} \cdot \frac{3}{38} \cdot \left[1 - \left(\frac{\xi_1}{\xi}\right)^a\right] \frac{\frac{1}{2}v_{\rm turb}^2}{\varepsilon_{\rm B,1}}$

Dynamo growth term is dominant when $1/2 v_{\rm turb}^2/\varepsilon_{\rm B,1} \gg 1$

$$a = 4/57$$

$\phi_{\rm ff}$: ratio between the collapse time and free-fall time

The quantities with the subscript ‘1’ denote the values at the onset of the non-linear stage.

- ✓ **However, these studies assumed isothermal EoS ($\gamma_{\rm eff} = 1.0$) and $v_{\rm turb}$ = Const in the non-linear phase, and Kolmogorov ($\propto k^{-5/3}$) spectrum.**

Aim of this study

- Generalizing the analytic/theoretical estimates expressing the evolution of the magnetic field in the collapsing gas clouds.

- Comparing the estimates with the results of numerical simulations with various conditions.

Generalize analytic/theoretical estimate

Kinematic stage
($\boldsymbol{\varepsilon_B \ll \varepsilon_{\rm turb}}$ on viscous scale)

$$B_{\rm kin} = B_0 \xi^{2/3} \exp\left(\frac{3}{8}\int \Gamma_\nu \, dt\right),$$

$$B_{\rm kin} = B_0 \xi^{2/3} \exp\left(C_\Gamma \int \Gamma_\nu \, dt\right),$$

where $\int \Gamma_\nu dt = (3/32)^{1/2} {\rm Re}^{\frac{1-\vartheta}{1+\vartheta}} \langle \mathcal{M}_{\rm turb} \rangle \int (1/t_{\rm ff}) dt$
$C_\Gamma = 0.0375$ (best-fit parameter)
$\langle \mathcal{M}_{\rm turb} \rangle$ and $\int (1/t_{\rm ff}) dt$ have $\gamma_{\rm eff}$ dependence.
(turbulence is amplified by gravitational collapse, SH+21)
ϑ:spectral index of turbulence (1/3 – 1/2)

Non-linear stage
($\boldsymbol{\varepsilon_B \sim \varepsilon_{\rm turb}}$ on viscous scale)

$$B_{\rm nl} = B_1 \mathcal{A}_1 \left(\frac{\xi}{\xi_1}\right)^{(1+a)/2}$$

$$\mathcal{A}_1^2 = 1 + 0.013 \frac{\phi_{\rm ff} \mathcal{M}_{\rm turb}}{a} \cdot \frac{3}{38} \cdot \left[1 - \left(\frac{\xi_1}{\xi}\right)^a\right] \frac{\frac{1}{2} v_{\rm turb}^2}{\varepsilon_{\rm B,1}}$$

$$B_{\rm nl} = B_1 \mathcal{A}_{\gamma,1} \left(\frac{\xi}{\xi_1}\right)^{(1+a_\gamma)/2}$$

$$\mathcal{A}_{\gamma,1}^2 = 1 + \frac{\phi_{\rm ff} \mathcal{M}_{\rm sat}}{\sqrt{6}\pi \left(1 - \frac{4}{19}\gamma_{\rm eff} - \frac{41}{57}\right)} \cdot \frac{3}{38} \cdot \left[1 - \left(\frac{\xi}{\xi_1}\right)^{\left(\frac{4}{19}\gamma_{\rm eff} - 1 + \frac{41}{57}\right)}\right] \frac{\frac{1}{2} v_{\rm sat.1}^2}{\varepsilon_{\rm B,1}}$$

$$a_\gamma = \frac{15}{19}\gamma_{\rm eff} - \frac{41}{57}, \quad \phi_{\rm ff} = \left(\frac{32\alpha(0)}{12\pi^2}\right)^{1/2}, \ \alpha(0) = 4\pi G t^2 \rho$$

($v_{\rm sat}$ and $\mathcal{M}_{\rm sat}$ can be estimated solely by $\gamma_{\rm eff}$, SH+22)

Estimate for the transition between 2 stages

At the onset of the non-linear stage, the energies on the viscous scale are the same (ε_ν).

Assuming the characteristic spectra for the magnetic and turbulent energies, we can obtain

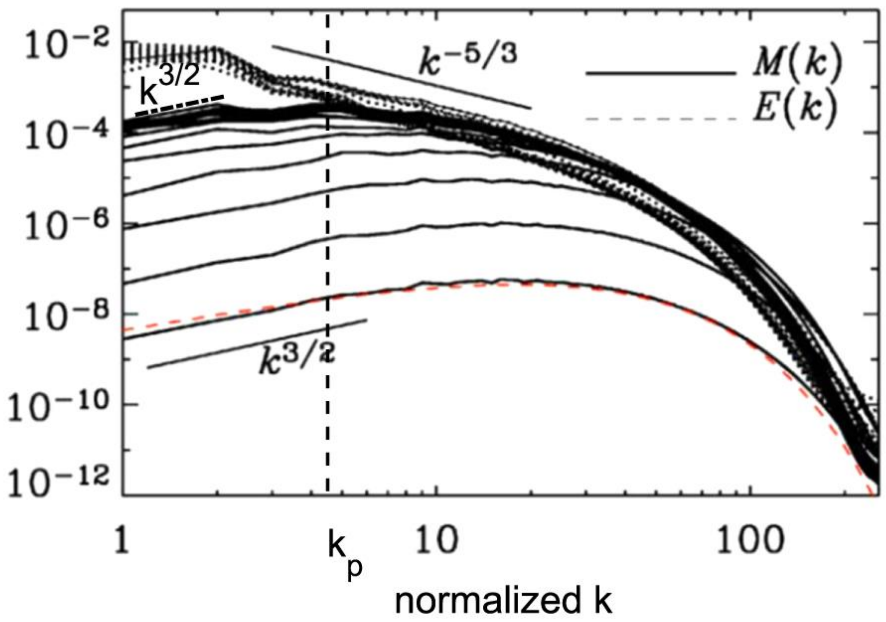
$$R_{\mathrm{nl},1} = \frac{\varepsilon_{\mathrm{B}}}{\varepsilon_{\mathrm{turb}}} = \frac{\int_{k_{\mathrm{J}}}^{k_\nu} \varepsilon_\nu \left(\frac{k}{k_\nu}\right)^{\frac{3}{2}} dk}{\int_{k_{\mathrm{J}}}^{k_\nu} \varepsilon_\nu \left(\frac{k}{k_\nu}\right)^{-\frac{5}{3}} dk}$$

Normalizing the wavenumber by k_{J},

$$R_{\mathrm{nl},1} = \frac{4}{15} N_{\mathrm{J}}^{-19/6} \frac{N_{\mathrm{J}}^{5/2} - 1}{1 - N_{\mathrm{J}}^{-2/3}}$$

$k_\nu / k_{\mathrm{J}} = \mathrm{N_J}$ (N_{J}: Jeans parameter)

For $N_{\mathrm{J}} = 128$, $R_{\mathrm{nl},1} \simeq 0.011$



Brandenburg&Subramanian 05

Numerical setup

- Using Athena++ 2nd order in time and space

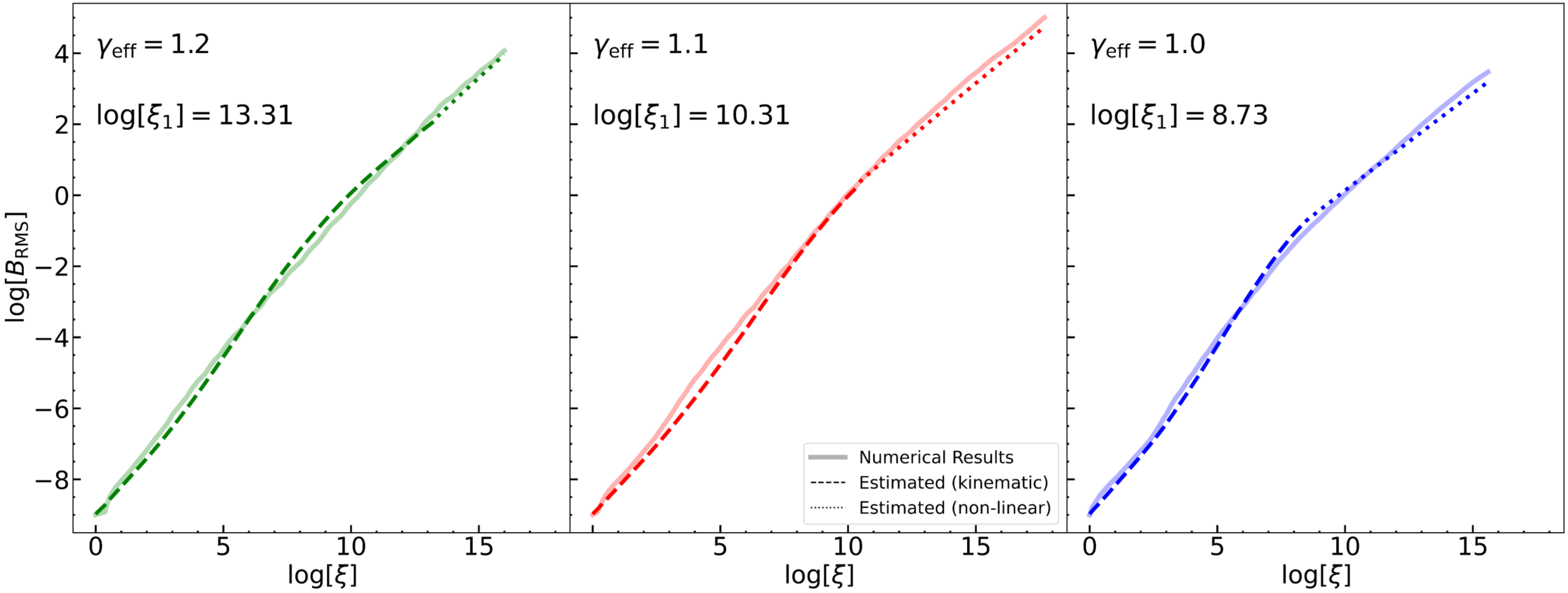
HLLD Riemann solver, CT method for div$B$, multigrid method for Poisson's equation

- Base grid 512^3. Resolve L_J by 128 cells + reso. study
- Barotropic EoS with $\gamma = 1.1, 1.2, 1.0$
- Bonnor-Ebert Sphere with

$$\rho_{\mathrm{peak},0} = 4.65 \times 10^{-20}\ \mathrm{g/cc},\ 200\ \mathrm{K},\ M_{\mathrm{cloud}} = 1700\ \mathrm{M}_\odot$$

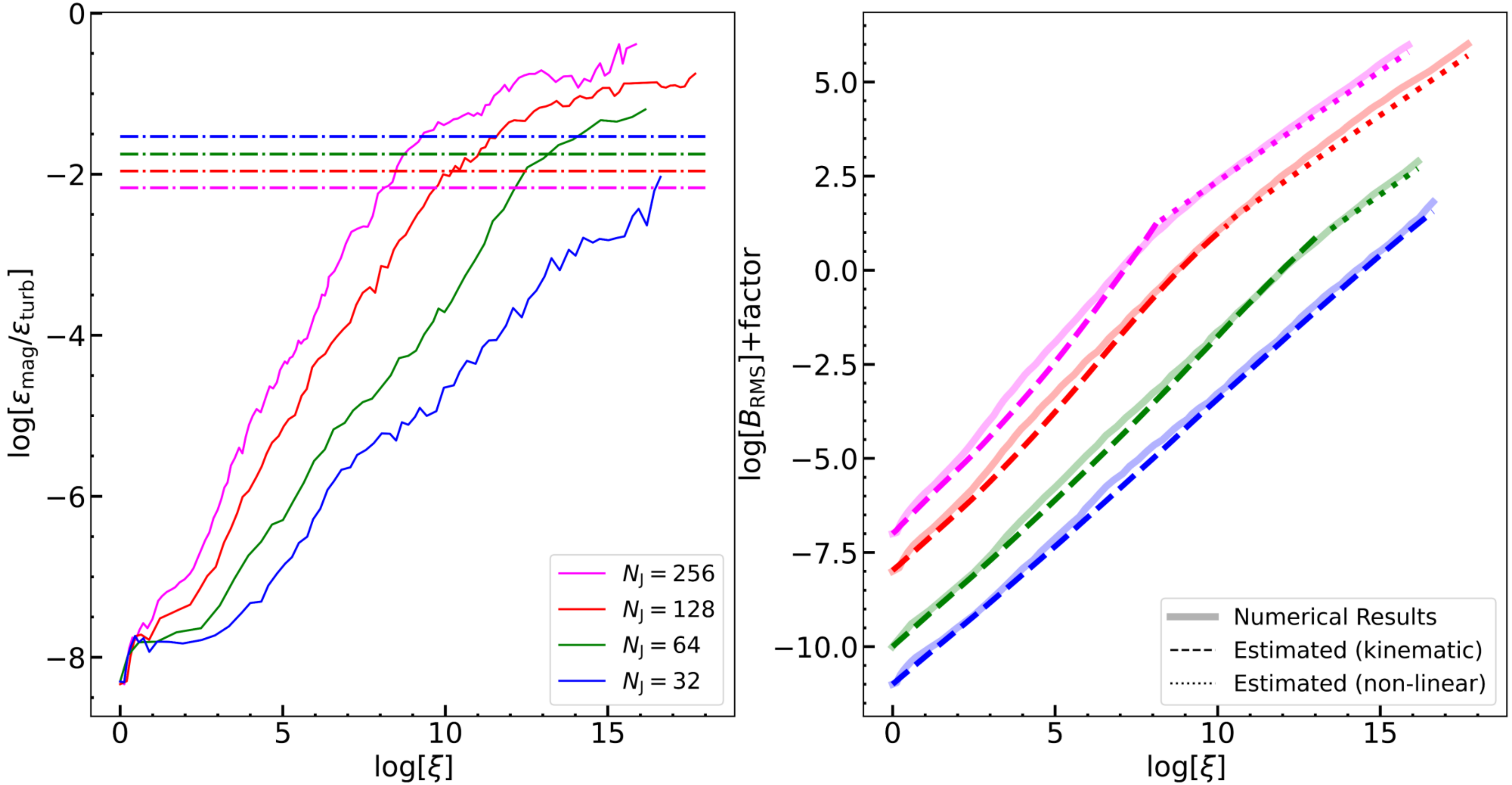
- Initial turbulence $\mathcal{M}_0 = 0.5$, $E_0(k) \propto k^{-2}$
- Initial B-field $B_0 = 10^{-9}$ G, uniform along z-axis
- Box size: $5 \times$ initial core radius with periodic boundary

Comparison with the estimates (1/2)



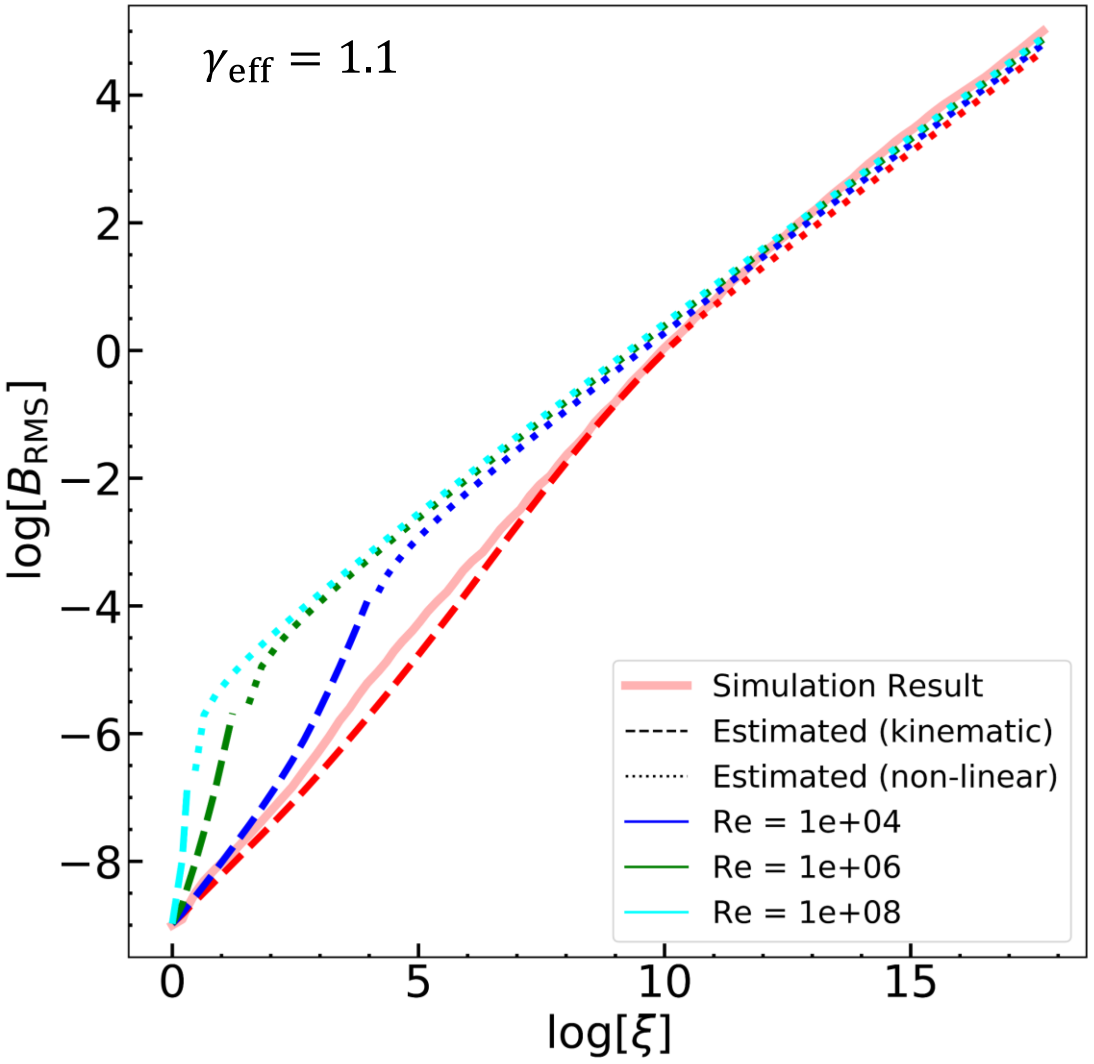
- ✓ The magnetic field strengths increase by 10^{12} -- 10^{15} orders of magnitude (3% -- 100% equipartition).
- ✓ Overall growth can be well-reproduced by the analytic estimate within a factor of a few.

Comparison with the estimates (2/2)



✓ **The growth in both stages can be well-reproduced in all resolutions.**

Higher Reynolds number cases



Dashed: $B_{\rm kin} = B_0 \xi^{2/3} \exp\left(C_\Gamma \int \Gamma_\nu \, dt\right)$

$$\int \Gamma_\nu dt = (3/32)^{1/2} {\rm Re}^{\frac{1-\vartheta}{1+\vartheta}} \langle \mathcal{M}_{\rm turb} \rangle \int (1/t_{\rm ff}) dt$$

Dotted: $B_{\rm nl} = B_1 \mathcal{A}_{\gamma,1} \left(\frac{\xi}{\xi_1}\right)^{(1+a_\gamma)/2}$

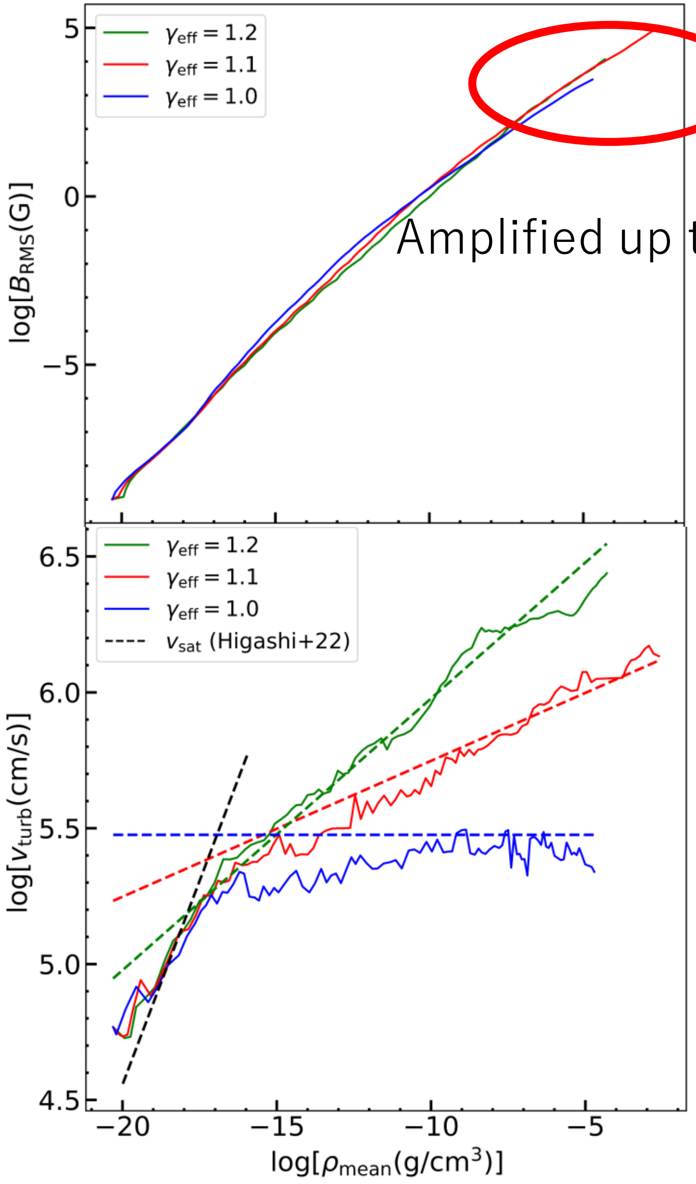
$$R_{\rm nl,1} = \frac{4}{15} N_{\rm J}^{-19/6} \frac{N_{\rm J}^{5/2} - 1}{1 - N_{\rm J}^{-2/3}} \qquad {\rm Re} \simeq N_{\rm J}^{4/3}$$

- ✓ Magnetic field strengths are converged even in higher Reynolds numbers.
- ✓ The strong magnetic field is a general property in the first-star formation.

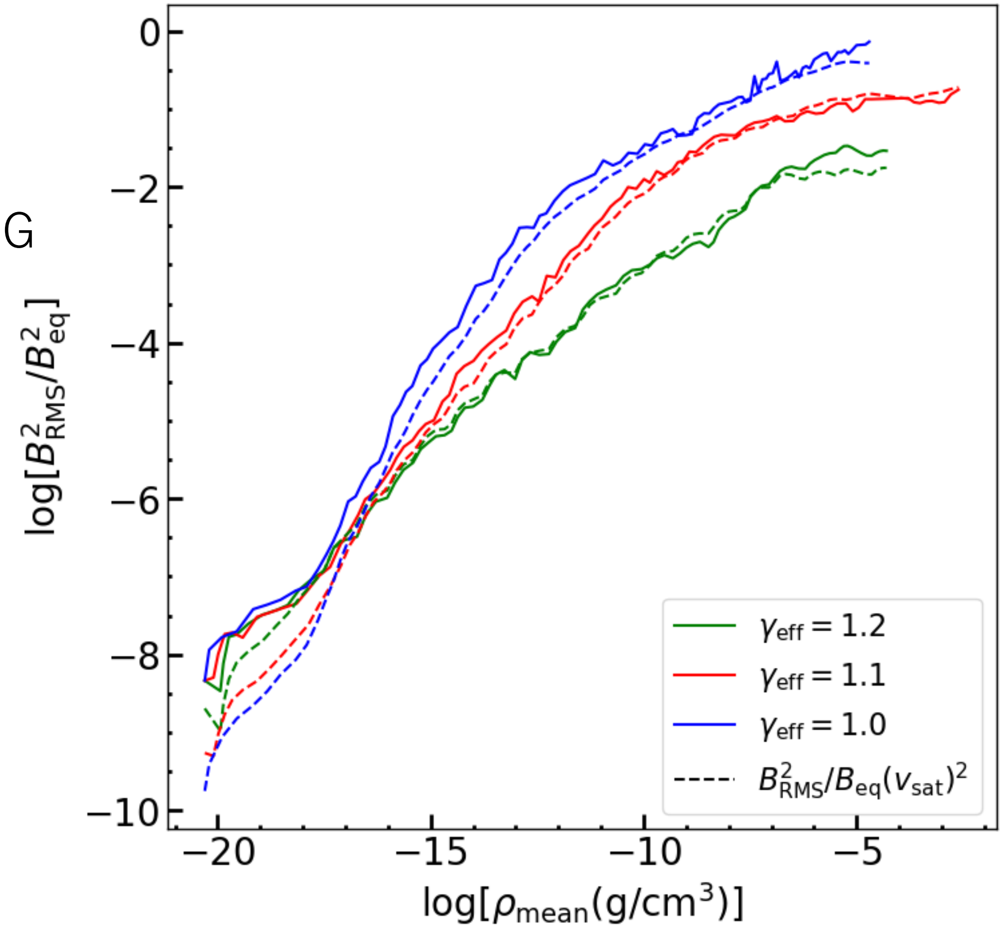
Summary

- The initially weak magnetic field in the early universe can be amplified 10^{12}--10^{15} orders of magnitude during the collapse phase.
- We generalize the earlier analytic/theoretical estimates expressing the evolution of the magnetic field in the collapsing gas clouds and compare them with the simulation results.
- Our generalized estimates can well-reproduce simulation results for various conditions and indicate a strong magnetic field is a general property in the first-star formation.
- The magnetic effects should be considered in the first-star formation.

Overall evolutions of the fields



Amplified up to $10^3 \sim 10^5$G



$\eta = 0.42$: energy dissipation coefficient
$\alpha(0)$: coefficient of collapse speed, depending on $\gamma_{\rm eff}$
(SH+22)

$$v_{\rm sat} = \frac{5-3\gamma_{\rm eff}}{3\eta}\frac{2\pi c_s}{\sqrt{\alpha(0)}}$$

$$B_{\rm eq} = \sqrt{4\pi\rho}v_{\rm turb}$$

Spectra evolution